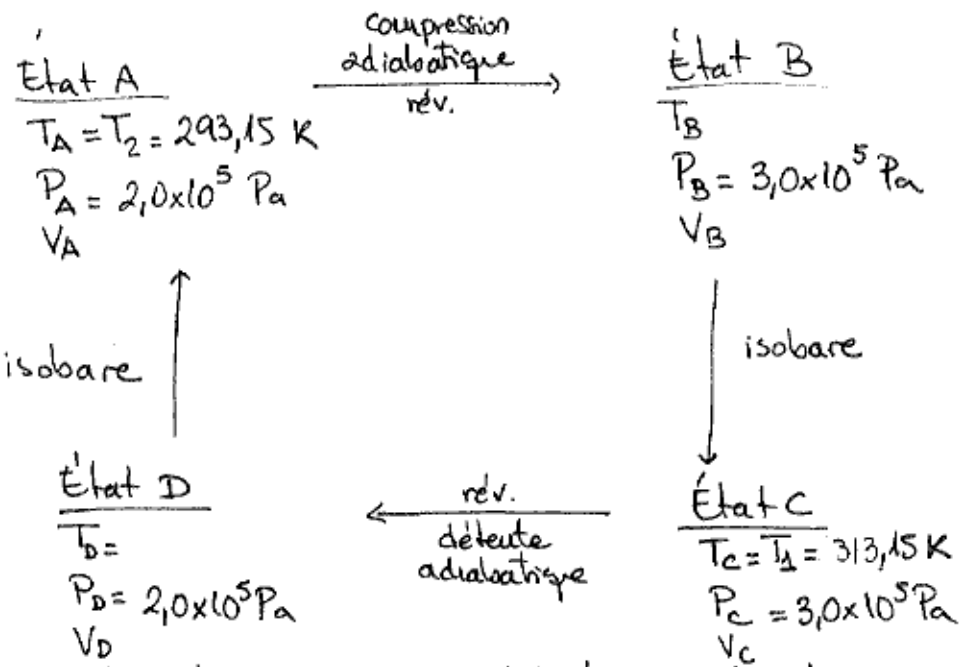


Exercice 1

Température du local  $T_2 = \theta_2 + 273,15 = 293,15 \text{ K}$

Température extérieure  $T_1 = \theta_1 + 273,15 = 313,15 \text{ K}$



① Transformation A → B adiabatique réversible

$$P \cdot V^\gamma = \text{cte} \Rightarrow P^{1-\gamma} T^\gamma = \text{cte}$$

$$P_A^{1-\gamma} \cdot T_A^\gamma = P_B^{1-\gamma} T_B^\gamma$$

$$T_B = T_A \left( \frac{P_A}{P_B} \right)^{\frac{1-\gamma}{\gamma}}$$

$$T_B = 344,77 \text{ K} \text{ soit } \underline{345 \text{ K}}$$

Transformation C → D adiabatique réversible

$$P^{1-\gamma} T = \text{cte}$$

$$P_C^{1-\gamma} T_C = P_D^{1-\gamma} T_D$$

$$T_D = T_C \left( \frac{P_C}{P_D} \right)^{\frac{1-\gamma}{\gamma}}$$

$$T_D = 266,27 \text{ K} \text{ soit } \underline{266 \text{ K}}$$

② Pour tracer l'allure du diagramme de Clapeyron on doit établir la relation entre les volumes  $V_A$ ,  $V_B$ ,  $V_C$  et  $V_D$

Transformation A → B adiabatique

$$P_A V_A^\gamma = P_B V_B^\gamma$$

$$V_B = V_A \left( \frac{P_A}{P_B} \right)^{1/\gamma} \quad V_B \approx 0,78 V_A$$

Transformation B → C isobare

$$\frac{T_B}{V_B} = \frac{T_C}{V_C}$$

$$V_C = V_B \frac{T_C}{T_B} = V_A \left( \frac{P_A}{P_B} \right)^{1/\gamma} \frac{T_C}{T_B}$$

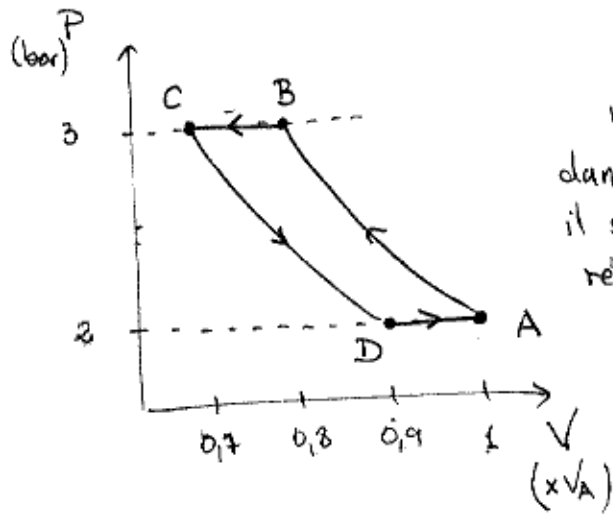
$$V_C \approx 0,71 V_A$$

Transformation  $C \rightarrow D$  adiabatique réversible

$$P_C V_C^\gamma = P_D V_D^\gamma$$

$$V_D = V_C \left( \frac{P_C}{P_D} \right)^{1/\gamma}$$

$$V_D = 0,91 V_A$$



Le cycle est décrit dans le sens trigonométrique il s'agit donc d'un cycle récepteur.

Le travail total reçu par le fluide est positif  
Il s'agit d'un cycle récepteur

③ Les chaleurs  $Q_1$  et  $Q_2$  échangées correspondent aux transformations  $B \rightarrow C$  ( $Q_1$ ) et  $D \rightarrow A$  ( $Q_2$ )

• Calcul de  $Q_1 = Q_{B \rightarrow C}$

3 4

Transformation isobare

$$Q_{B \rightarrow C} = m c_p' \Delta T = m c_p' (T_C - T_B)$$

$$Q_1 = Q_{B \rightarrow C} = -1,66 \times 10^5 \text{ J}$$

• Calcul de  $Q_2 = Q_{D \rightarrow A}$

Transformation isobare

$$Q_{D \rightarrow A} = m c_p' \Delta T = m c_p' (T_A - T_D)$$

$$Q_2 = Q_{D \rightarrow A} = 1,41 \times 10^5 \text{ J}$$

④ Transformation  $A \rightarrow B$  adiabatique réversible

$$W_{A \rightarrow B} = \int_A^B \delta W = \int_A^B -P_e dV = \int_A^B -P dV = \int_{V_A}^{V_B} -A \frac{dV}{V^\gamma} = -A \int_{V_A}^{V_B} \frac{dV}{V^\gamma} = -A \left. \frac{V^{-\gamma+1}}{-\gamma+1} \right|_{V_A}^{V_B}$$

$$= - \left. \frac{P \cdot V^\gamma \cdot V^{-\gamma+1}}{-\gamma+1} \right|_{V_A}^{V_B} = - \left. \frac{P \cdot V}{-\gamma+1} \right|_{V_A}^{V_B} =$$

$$= + \left. \frac{R n T}{\gamma-1} \right|_{T_A}^{T_B} = \frac{R n}{\gamma-1} (T_B - T_A)$$

$$n = \frac{m}{M} = 250 \text{ mol d'hélium}$$

$$W_{A \rightarrow B} = +1,61 \times 10^5 \text{ J}$$

Transformation B  $\rightarrow$  C isobare

$$W_{B \rightarrow C} = \int_B^C \delta W = - \int_B^C P dV = -P_B \int_B^C dV = -P_B \cdot V \Big|_B^C$$

$$= -P(V_C - V_B) = Rn(T_B - T_C) = 6,57 \times 10^4 \text{ J}$$

Transformation C  $\rightarrow$  D adiabatique

$$W_{C \rightarrow D} = \frac{Rn}{\gamma - 1} (T_D - T_C) = -1,46 \times 10^5 \text{ J}$$

Transformation D  $\rightarrow$  A isobare

$$W_{D \rightarrow A} = Rn(T_D - T_A) = -5,59 \times 10^4 \text{ J}$$

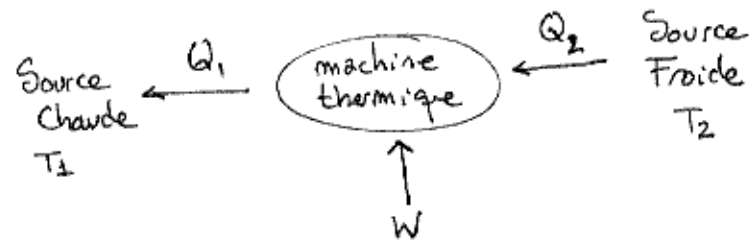
$$W_{\text{cycle}} = 2,46 \times 10^4 \text{ J}$$

$$\textcircled{5} \quad \text{COP}_{\text{Froid}} = \frac{Q_2}{W}$$

$$\text{COP}_{\text{Froid}} = 5,73$$

$\textcircled{6}$  Dans le cas d'une machine frigorifique de Carnot

5 6



D'après le 1<sup>er</sup> principe, pour le cycle :

$$Q_1 + Q_2 + W = 0$$

Selon le 2<sup>ème</sup> principe, pour un cycle

$$\oint \frac{\delta Q}{T} \leq 0$$

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} \leq 0$$

Pour un cycle réversible

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} = 0 \Rightarrow \frac{Q_1}{T_1} = -\frac{Q_2}{T_2}$$

Le coefficient de performance d'une machine frigorifique

$$\text{COP}_{\text{Froid}} = \frac{Q_2}{W}$$

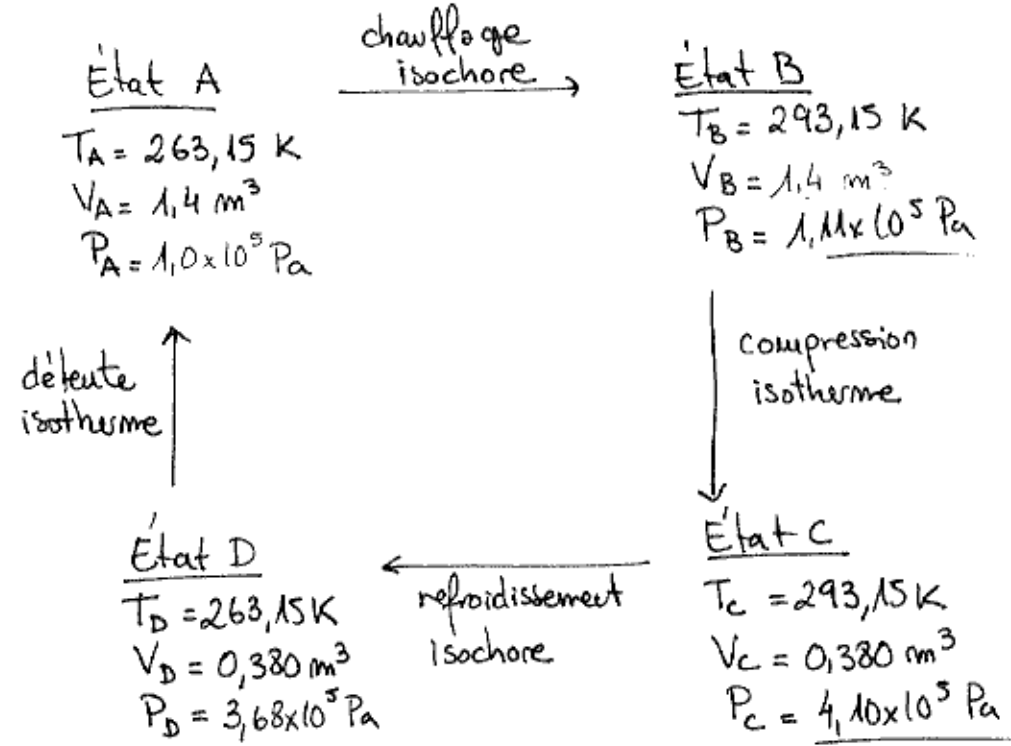
$$\text{COP}_{\text{Froid}} = -\frac{Q_2}{Q_1 + Q_2} = -\frac{Q_2}{-\frac{T_1}{T_2} Q_2 + Q_2} =$$

$$= \frac{1}{\frac{T_1}{T_2} - 1} = \frac{1}{\frac{T_1 - T_2}{T_2}} = \frac{T_2}{T_1 - T_2}$$

$$\text{COP}_{\text{Froid}} = 14,7$$

7 8

**Exercice 2**



Transformation A → B isochore

$$\frac{P_A}{T_A} = \frac{P_B}{T_B} \quad P_B = P_A \frac{T_B}{T_A} = 1,11 \times 10^5 \text{ Pa}$$

Transformation B → C isotherme

$$P_B V_B = P_C V_C \quad P_C = P_B \frac{V_B}{V_C}$$

$$P_C = 4,10 \times 10^5 \text{ Pa}$$

Transformation D → C isotherme

$$P_A V_A = P_D V_D$$

$$P_B = P_A \frac{V_A}{V_D} = 3,68 \times 10^5 \text{ Pa}$$

① Transformation A → B isochore (SW=0)

$$Q_{A \rightarrow B} = \int_A^B \delta Q = \int_A^B du = \int_A^B n c_v dT =$$

$$= n c_v (T_B - T_A) = \frac{P_A V_A}{R T_A} c_v (T_B - T_A)$$

$$Q_{A \rightarrow B} = 3,99 \times 10^4 \text{ J}$$

Transformation isotherme B → C (du=0)

$$du = \delta Q + \delta W \quad \delta Q = -\delta W$$

$$Q_{B \rightarrow C} = \int_B^C \delta Q = - \int_B^C \delta W = - \int_B^C P dV =$$

↑  
P.V = cte = P<sub>B</sub>V<sub>B</sub>

$$= - P_B V_B \int_{V_B}^{V_C} \frac{dV}{V} = - P_B V_B \ln \frac{V_C}{V_B}$$

$$Q_{B \rightarrow C} = -2,03 \times 10^5 \text{ J}$$

Transformation isochore C → D

$$Q_{C \rightarrow D} = \int_C^D \delta Q = \int_C^D du = \int_C^D n c_v dT =$$

$$n c_v (T_D - T_C) = \frac{P_A V_A}{R T_A} c_v (T_D - T_C)$$

$$Q_{C \rightarrow D} = -3,99 \times 10^4 \text{ J}$$

Transformation isotherme D  $\rightarrow$  A

$$Q_{D \rightarrow A} = \int_D^A \delta Q = - \int_D^A \delta W = + \int_D^A P dV =$$

$$= + P_A V_A \int_D^A \frac{dV}{V} = P_A V_A \ln \frac{V_A}{V_D}$$

$$Q_{D \rightarrow A} = 1,83 \times 10^5 \text{ J}$$

② Lors de transformations isochores  $W=0$ :

$$W_{A \rightarrow B} = W_{C \rightarrow D} = 0$$

Pour les transformations isothermes:  $W = -Q$

$$W_{B \rightarrow C} = -Q_{B \rightarrow C} = 2,03 \times 10^5 \text{ J}$$

$$W_{D \rightarrow A} = -Q_{D \rightarrow A} = -1,83 \times 10^5 \text{ J}$$

$$W_{\text{cycle}} = W_{A \rightarrow B} + W_{B \rightarrow C} + W_{C \rightarrow D} + W_{D \rightarrow A} =$$

$$= R_n (T_A - T_B) \ln \frac{V_C}{V_A} = 2,08 \times 10^4 \text{ J}$$

③ La seule chaleur qui peut être fournie au local est  $Q_{BC}$

$$\text{COP}_{\text{PAC}} = \frac{-Q_{BC}}{W_{\text{cycle}}}$$

9 10

$$W_{\text{cycle}} = -Q_{B \rightarrow C} - Q_{D \rightarrow A} = -P_B V_B \ln \frac{V_C}{V_B} -$$

$$P_A V_A \ln \frac{V_A}{V_D} = R_n (T_A - T_B) \ln \frac{V_C}{V_A}$$

$$V_A = V_B$$

$$V_C = V_D$$

$$\text{COP}_{\text{PAC}} = \frac{-R_n T_B \ln \frac{V_C}{V_A}}{R_n (T_B - T_A) \ln \frac{V_C}{V_A}}$$

$$\text{COP}_{\text{PAC}} = \frac{T_B}{T_B - T_A} = 9,77$$

### Exercice 3

$$① ds = \frac{\delta Q_r}{T}$$

Selon le 1<sup>er</sup> principe  $\delta Q_r = du - \delta W_r$

Pour un gaz parfait  $du = n c_v dT$

$$\delta Q_r = n c_v dT + P dV \Rightarrow ds = n c_v \frac{dT}{T} + \frac{P}{T} dV$$

A partir de l'équation des gaz parfaits

$$PV = R_n T$$

$$V dP + P dV = R_n dT$$

$$VdP + PdV = \frac{PV}{T} dT$$

$$\frac{dP}{P} + \frac{dV}{V} = \frac{dT}{T}$$

$$ds = n c_v \frac{dT}{T} + \frac{P}{T} dV = n c_v \frac{dT}{T} + \frac{Rn}{V} dV$$

$$= n c_v \left( \frac{dP}{P} + \frac{dV}{V} \right) + \frac{Rn}{V} dV =$$

$$= \frac{n c_v}{P} dP + n (c_v + R) \frac{dV}{V} = \frac{n c_v}{P} dP + \frac{n c_p}{V} dV$$

$$\gamma = \frac{c_p}{c_v}$$

$$R = c_p - c_v$$

$$c_p = \gamma c_v$$

$$R = \gamma c_v - c_v$$

$$(\gamma - 1) c_v = R \quad c_v = \frac{R}{\gamma - 1}$$

$$c_p = R + c_v = R + \frac{R}{\gamma - 1} = \frac{\gamma R}{\gamma - 1}$$

$$ds = \frac{n R}{\gamma - 1} \left( \frac{dP}{P} + \gamma \frac{dV}{V} \right)$$

$$S = \int ds = \frac{n R}{\gamma - 1} (\ln P + \gamma \ln V) + c =$$

$$= \frac{n R}{\gamma - 1} \ln PV^\gamma + c;$$

11

12

2 (a) Si la transformation est adiabatique réversible  $PV^\gamma = \text{cte}$

$$\Delta S = S_1 - S_0 = 0 \text{ J.K}^{-1}$$

(b) Pour une transformation isotherme  $P.V = \text{cte}$

$$ds = \frac{P}{T} dV = Rn \frac{dV}{V}$$

$$\Delta S_0' = \int_{V_0}^{V_1} Rn \frac{dV}{V} = Rn \ln \frac{V_1}{V_0} \quad \uparrow \quad Rn \ln \frac{P_0}{P_1}$$

$$P_1 V_1 = P_0 V_0 \quad \frac{V_1}{V_0} = \frac{P_0}{P_1}$$

$$\Delta S_0' = -13,4 \text{ J.K}^{-1}$$

### Exercice 4

(1) Voir exercice précédent

$$ds = \frac{n c_v}{T} dT + \frac{P}{T} dV = \frac{n c_v}{T} dT + \frac{Rn}{V} dV$$

Transformation isochore  $T_0 = 298 \text{ K} \rightarrow T_1 = 400 \text{ K}$

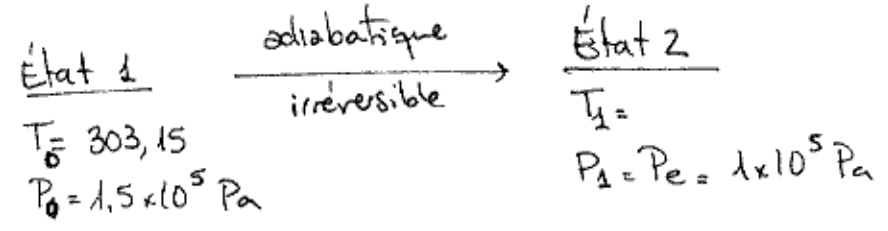
$$\Delta S_0' = \int_0^1 ds = \int_{T_0}^{T_1} \frac{n c_v}{T} dT = n \int_{T_0}^{T_1} \left( \frac{23,83}{T} + \frac{22,15 \times 10^{-3}}{T} \right) dT$$

$$= n \left[ 23,83 \ln \frac{T_1}{T_0} + 22,15 \times 10^{-3} (T_1 - T_0) \right] = \underline{46,4 \text{ J.K}^{-1}}$$

**Exercice 5**

13 14

$n = 100 \text{ mol}$



① Comme, il s'agit d'une adiabatique  $Q = 0$

$W = \Delta U$

$W = \int_0^1 -P_e dV = -P_e \int_{V_0}^{V_1} dV = -P_e (V_1 - V_0)$

$\Delta U_0^1 = \int_0^1 dU = \int_{T_0}^{T_1} n c_v dT = n c_v (T_1 - T_0)$

$W = -P_e (V_1 - V_0) = -P_e R n \left( \frac{T_1}{P_1} - \frac{T_0}{P_0} \right) =$   
 $= R n \left( T_0 \frac{P_1}{P_0} - T_1 \right)$

$n c_v (T_1 - T_0) = R n \left( T_0 \frac{P_1}{P_0} - T_1 \right)$

$(c_v + R) T_1 = \left( R \frac{P_1}{P_0} + c_v \right) T_0$

$T_1 = \frac{\left( R \frac{P_1}{P_0} + c_v \right) T_0}{(c_v + R)}$

$C_p - C_v = R$        $C_v = C_p - R$

$C_p = R + C_v$

$T_2 = T_0 \frac{R \frac{P_1}{P_0} + C_p - R}{C_p}$

$T_1 = T_0 \frac{R \left( \frac{P_1}{P_0} - 1 \right) + C_p}{C_p}$

$T_1 = 262,73 \text{ K}$

②  $ds = \frac{\delta Q_r}{T}$

On doit exprimer  $S$  en fonction de  $T$  et  $P$

$\delta Q_r = n c_v dT + P dV$

À partir de l'équation des gaz parfaits

$P dV + V dP = R n dT$

$\frac{dV}{V} + \frac{dP}{P} = \frac{dT}{T} \Rightarrow \frac{dV}{V} = \frac{dT}{T} - \frac{dP}{P}$

$ds = \frac{\delta Q_r}{T} = n c_v \frac{dT}{T} + \frac{P}{T} dV = n c_v \frac{dT}{T} + R n \frac{dV}{V}$

$ds = n c_v \frac{dT}{T} + R n \left( \frac{dT}{T} - \frac{dP}{P} \right) =$

$= n (c_v + R) \frac{dT}{T} + R n \frac{dP}{P} = n c_p \frac{dT}{T} - R n \frac{dP}{P}$

$$\Delta S_0' = \int_0^1 ds = \int_{T_0}^{T_1} n c_p \frac{dT}{T} - \int_{P_0}^{P_1} R n \frac{dP}{P} =$$

$$= n c_p \ln \frac{T_1}{T_0} - R n \ln \frac{P_1}{P_0}$$

$$\Delta S_0' = 39,67 \text{ J K}^{-1}$$

### Exercice 6

$$m = 2 \text{ Kg}$$

$$n = \frac{m}{M} = 62,418 \text{ mol}$$

$$\text{Transformation isobare} \Rightarrow \delta Q_r = dH = n c_p dT$$

$$\textcircled{1} ds = \frac{\delta Q_r}{T} = n c_p \frac{dT}{T}$$

$$\Delta S_0' = \int_0^1 ds = \int_0^1 n c_p \frac{dT}{T} = n \int_{T_0}^{T_1} \frac{(52,8 + 0,09T)}{T} dT$$

$$= n \left( \int_{T_0}^{T_1} \frac{52,8}{T} dT + \int_{T_0}^{T_1} 0,09 dT \right) = n \left( 52,8 \ln \frac{T_1}{T_0} + \right.$$

$$\left. 0,09 (T_1 - T_0) \right) = 568,3 \text{ J K}^{-1}$$

② Bilan entropique

$$\Delta S_0' = \int_0^1 \frac{\delta Q}{T_e} + \int_0^1 \delta \sigma$$

$$\int_0^1 \frac{\delta Q}{T_e} = \frac{1}{T_e} \int_0^1 \delta Q = \frac{1}{T_e} \int_0^1 n c_p dT =$$

$$= \frac{1}{T_e} n \int_{T_0}^{T_1} (52,8 + 0,09T) dT =$$

$$= \frac{n}{T_e} \left[ 52,8 (T_1 - T_0) + \frac{0,09}{2} (T_1^2 - T_0^2) \right]$$

$$\sigma = \Delta S_0' - \int_0^1 \frac{\delta Q}{T_e}$$

$$568,3 - 537,6 = \underline{30,68 \text{ J K}^{-1}}$$

③ Si l'échange de chaleur se réalise avec une source à 75°C ( $T_e = 348,15 \text{ K}$ )

$$\begin{aligned} \int_0^1 \frac{\delta Q}{T_e} &= \frac{n}{T_e} \left[ 52,8 (T_1 - T_0) + \frac{0,09}{2} (T_1^2 - T_0^2) \right] \\ &= 506,757 \text{ J K}^{-1} \end{aligned}$$

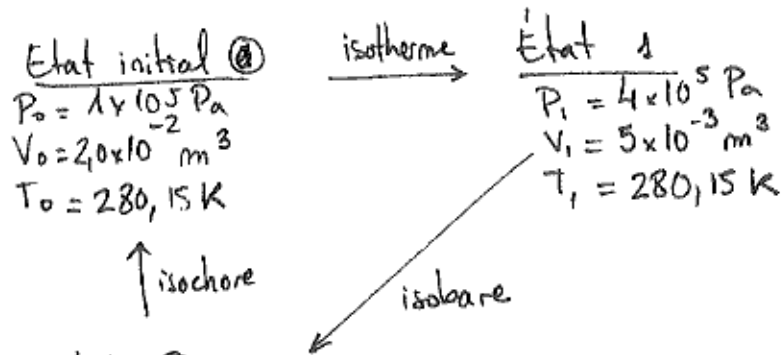
$$\sigma = \Delta S_0' - \int_0^1 \frac{\delta Q}{T_e} = \underline{61,56 \text{ J K}^{-1}}$$

Plus l'écart de température est élevé plus la production d'entropie est importante car l'irréversibilité de l'échange augmente.



# Exercice 7

$m = 25 \text{ g}$        $n = \frac{m}{M} = 0,86311 \text{ mol}$



Etat ②

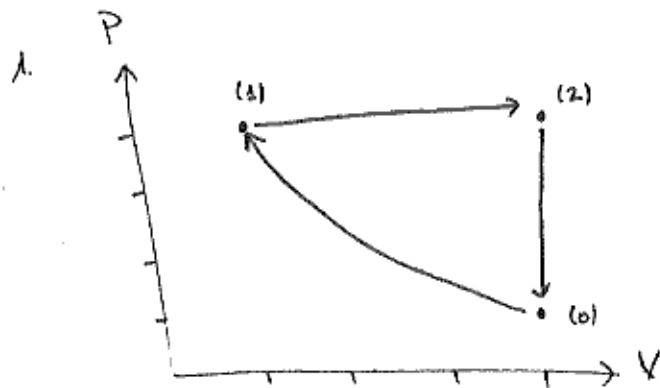
$P_2 = P_1 = 4 \times 10^5 \text{ Pa}$   
 $V_2 = 2,0 \times 10^{-2} \text{ m}^3$   
 $T_2 = 1120,6 \text{ K}$

Calcul de  $V_1$

0 → 1 Isotherme  
 $P_0 V_0 = P_1 V_1$        $V_1 = \frac{P_0 V_0}{P_1}$

Calcul de  $T_2$

1 → 2 Isobare  
 $\frac{V_1}{T_1} = \frac{V_2}{T_2}$        $T_2 = \frac{V_2}{V_1} T_1$



17 18 2.

(0) → (0) Transformation isotherme

$$ds = n \frac{C_v dt}{T} + P \frac{dV}{T}$$

$$\Delta S_{0 \rightarrow 1} = \int_0^1 n c_v \frac{dT}{T} + \int_0^1 R_n \frac{dV}{V}$$

$$= R_n \ln \frac{V_1}{V_0} = -9,949 \text{ J} \cdot \text{K}^{-1}$$

(1) → (2) isobare

$$dQ = dH = n c_p dt$$

$$\Delta S_{1 \rightarrow 2} = \int_1^2 \frac{n c_p}{T} dt = n c_p \ln \frac{T_2}{T_1} = 34,82 \text{ J} \cdot \text{K}^{-1}$$

(2) → (0) isochore

$$\Delta S_{0 \rightarrow 2} = \int_2^0 ds = \int_2^0 \frac{n c_v dt}{T}$$

$$\Delta S = n c_v \ln \frac{T_0}{T_2} = -24,87 \text{ J} \cdot \text{K}^{-1}$$

3.  $\Delta S_{\text{total}} = \Delta S_{(0) \rightarrow (1)} + \Delta S_{(1) \rightarrow (2)} + \Delta S_{(2) \rightarrow (0)} = 0 \text{ J} \cdot \text{K}^{-1}$