Correction type exercices séance 3

Exercice 1

1. La centrale étant considérée comme un moteur thermique :

- la source chaude fournit de la chaleur au fluide $Q_1 > 0$.
- la source froide reçoit de la chaleur du fluide $Q_2 < 0$.
- le système fourni du travail $W < 0$.

2. $\eta = \frac{-W}{Q_1}$ $\quad \eta = \frac{\text{l'énergie utile}}{\text{énergie à payer}}$

3. $\eta_c = 1 - \frac{T_2}{T_1}$ $\quad T_1 = \theta_1 + 273{,}15 = 600{,}15\,K$
$T_2 = \theta_2 + 273{,}15 = 288{,}15\,K$

$\eta_c = 1 - \frac{288.15}{600{,}15} = 0{,}52$

4. Efficacité $E_{moteur} = \frac{\eta}{\eta_c}$ $\quad \eta = E_{moteur} \times \eta_c$

$\eta = 0{,}58 \times 0{,}52 = 0{,}3016$

Le rendement de la centrale es de 30%

5. $\eta = \frac{-\dot{W}}{\dot{Q}_1}$ $\quad |\dot{W}| = \eta \dot{Q}_1 = 9{,}00 \times 10^8\,W$

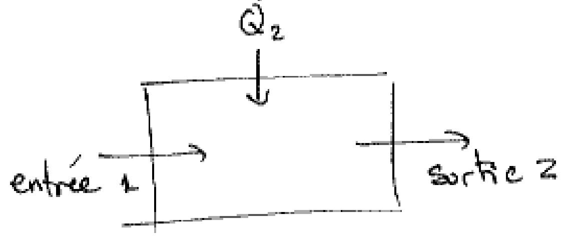
$\dot{W} = -9{,}00 \times 10^8\,W$

6. D'après le 1er principe de la thermodynamique

$$\dot{Q}_1 + \dot{Q}_2 + \dot{W} = 0$$

$$\dot{Q}_2 = -\dot{W} - \dot{Q}_1 = 9{,}00 \times 10^8 - 3000 \times 10^6 = -2{,}095 \times 10^9\,J$$

7. On peut considérer le fleuve comme un système ouvert stationnaire



$\dot{m}\,h_2 - \dot{m}\,h_1 = \dot{Q}_2$ $\quad \dot{m} = \dot{v} \times \rho$

$\dot{m}\,(h_2 - h_1) = \dot{Q}_2$

$\dot{v} \times \rho\,(h_2 - h_1) = \dot{Q}_2$ $\quad \dot{v} = \frac{\dot{Q}_2}{\rho\,(c'_{eau}\,\Delta T)}$

Si $\Delta T = 2\,°C$ $\quad \Delta h =$

$$\dot{v} = 250\ m^3 . s^{-1}$$

Exercice 2

Cycle de Carnot ABCDA

État A $\xrightarrow{\text{isotherme}}$ État B

$T_A = 300\,K$, $P_A = 1\,bar$, V_A

$T_B = 300\,K$, $P_B = 3\,bar$, $V_B = \frac{V_A}{3}$

État B → État C : adiabatique

État D → État A : adiabatique

État D $\xleftarrow{\text{isotherme}}$ État C

$T_D = 411\,K$, P_D, V_D

$T_C = 411\,K$, $P_C = 9\,bar$, V_C

On considère 1 Kg d'air
On calcule la quantité de matière

$$n = \frac{m}{M} = \frac{1000}{29{,}1} = 34{,}4 \text{ mol}$$

$$V_A = \frac{RnT}{P_A} = 8{,}57 \times 10^{-1}\,m^3$$

$A \to B$ Transformation isotherme.

$$P_A V_A = P_B V_B \longrightarrow \frac{V_B}{V_A} = \frac{P_A}{P_B}$$

$$V_B = \frac{P_A V_A}{P_B} = 0{,}286\,m^3$$

$B \to C$ Transformation adiabatique

$$PV^{\gamma} = cte$$

$$V = \frac{RnT}{P} \qquad P\cdot\left(\frac{T}{P}\right)^{\gamma} = cte$$

$$T^{\gamma}\cdot P^{1-\gamma} = cte \Rightarrow T\,P^{\frac{1-\gamma}{\gamma}} = cte$$

$$T_B P_B^{\frac{1-\gamma}{\gamma}} = T_C P_C^{\frac{1-\gamma}{\gamma}}$$

$$(1) \quad T_C = T_B \left(\frac{P_B}{P_C}\right)^{\frac{1-\gamma}{\gamma}} = 411\,K$$

$D \to A$: transformation adiabatique

$$T_A P_A^{\frac{1-\gamma}{\gamma}} = T_D P_D^{\frac{1-\gamma}{\gamma}}$$

$$(2) \quad T_D = T_A \left(\frac{P_A}{P_D}\right)^{\frac{1-\gamma}{\gamma}}$$

$$\frac{(1)}{(2)} \quad \left(\frac{P_B}{P_A}\right)^{\frac{1-\gamma}{\gamma}} \quad \left(\frac{P_C}{P_D}\right)^{\frac{1-\gamma}{\gamma}}$$

$$\frac{P_B}{P_A} = \frac{P_C}{P_D} \qquad P_D = \frac{P_C P_A}{P_B} = 3\,bar$$

Au cours d'une isotherme $T = cte$ $dU = 0$

$$\delta Q = -\delta W = -(-P_e\,dV) \underset{\text{réversible}}{=} +P\,dV$$

$A \to B$

$$Q_{A\to B} = +\int_A^B P\,dV = RnT_A \int_{V_A}^{V_B} \frac{dV}{V} =$$

$$RnT_A \ln \frac{V_B}{V_A} = RnT_A \ln \frac{P_A}{P_B} = -9{,}42 \times 10^{4}\,J$$

Isotherme $C \to D$

$$Q_{C\to D} = \int_C^D \delta Q = \int_C^D -\delta W = +\int_{V_C}^{V_D} P\,dV = R n T_C \int_{V_C}^{V_D} \frac{dV}{V} ;$$

$$= R n T \ln \frac{V_D}{V_C} = R n T_C \ln \frac{P_C}{P_D} = \underline{1{,}29 \times 10^5 \text{ J}}$$

1er principe

$$W_{cycle} + Q_{cycle} = 0$$

$$W_{cycle} = -Q_{cycle} = Q_{A\to B} + Q_{C\to D} = -3{,}48 \times 10^4 \text{ J}$$

$$\eta = \frac{-W}{Q_1} = \underline{\underline{0{,}27}}$$

b)

$$\eta = 1 - \frac{T_{sf}}{T_{sc}} = \underline{0{,}27}$$

2. $dS = \dfrac{\delta Q_r}{T}$

$A \to B$ $\quad \Delta S_A^B = \int_A^B dS = \int_A^B \frac{\delta Q_r}{T_A} = \frac{1}{T} \int_A^B \delta Q_r =$

$$= \frac{Q_{A\to B}}{T_A} = -313{,}9 \text{ J}$$

$C \to D$

$$\Delta S_C^D = \int_C^D \frac{\delta Q_r}{T} = \frac{1}{T_C} \int_C^D \delta Q_r = \frac{Q_{C\to D}}{T_C} = \underline{313{,}9 \text{ J}}$$

$$\Delta S_{cycle} = 0$$

Exercice 3

État 1 $\xrightarrow{\text{isotherme}}$ État 2

État 1: $P_1 = 1 \times 10^5$ Pa ; $T_1 = 350$ K ; $V_1 = 1\ m^3$

État 2: $P_2 = 8 \times 10^5$ Pa ; $T_2 = T_1 = 350$ K ; $V_2 = 0{,}125\ m^3$

État 2 $\xrightarrow{\text{isobare}}$ État 3

État 3: $P_3 = P_2 = 8 \times 10^5$ Pa ; $T_3 = 1400$ K ; $V_3 = 0{,}5\ m^3$

État 3 $\xrightarrow{\text{adiabatique}}$ État 4

État 4: $P_4 = P_1 = 1 \times 10^5$ Pa ; $T_4 = 772{,}8$ K ; $V_4 = 2{,}208\ m^3$

État 4 $\xrightarrow{\text{isobare}}$ État 1

1. $C_p - C_v = R$ (Relation de Mayer)

$\gamma = \dfrac{C_p}{C_v}$ $\qquad C_v = \dfrac{C_p}{\gamma}$ $\qquad C_p - \dfrac{C_p}{\gamma} = R$

$$\frac{\gamma C_p - C_p}{\gamma} = R$$

$$C_p = \frac{\gamma R}{\gamma - 1} = 29{,}10 \text{ J K}^{-1}\text{mol}^{-1}$$

$$c_p' = \frac{C_p}{M} = 1000\ \mathrm{J\,Kg^{-1}\,K^{-1}}$$

b) État 1

$$V_1 = \frac{RnT_1}{P_1} = 1\ m^3$$

État 2 $\quad 1 \to 2$ isotherme

$$P_1 V_1 = P_2 V_2$$

$$V_2 = \frac{P_1 V_1}{P_2} = 0{,}125\ m^3$$

État 3

$2 \to 3$ isobare $\quad \frac{V}{T} = cte$

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \qquad V_3 = V_2 \frac{T_3}{T_2} = 0{,}5\ m^3$$

État 4

$3 \to 4$ adiabatique

$$PV^\gamma = cte$$

$$P_3 V_3^\gamma = P_4 V_4^\gamma \qquad \frac{V_4}{V_3} = \left(\frac{P_3}{P_4}\right)^{1/\gamma}$$

$$V_4 = V_3 \left(\frac{P_3}{P_4}\right)^{1/\gamma} = 2{,}208\ m^3$$

7 8

Isobare $4 \to 1$

$$\frac{V_1}{T_1} = \frac{V_4}{T_4} \qquad T_4 = \frac{V_4}{V_1} T_1 = 772{,}8\ K$$

$$\eta = \frac{\text{l'énergie utile}}{\text{énergie à payer}}$$

$$\eta = \frac{|W|}{Q_{sc}} \leftarrow \text{chaleur reçue de la source chaude}$$

$$W = W_{1\to2} + W_{2\to3} + W_{3\to4} + W_{4\to1}$$

$1 \to 2$ isotherme

$$W = \int \delta W = -\int_1^2 P\,dV = -RnT_1 \int_1^2 \frac{dV}{V} =$$

$$= -RnT_1 \ln \frac{V_2}{V_1} = -P_1 V_1 \ln \frac{P_2}{P_1} = +2{,}080 \times 10^5\ J$$

$2 \to 3$ isobare

$$W = \int \delta W = -P_2 \int_2^3 dV = -P_2 (V_3 - V_2) = -3{,}00 \times 10^5\ J$$

$3 \to 4$ adiabatique

$$W = \int \delta W = -\int P\,dV = -A \int \frac{dV}{V^\gamma} = -A \left.\frac{V^{-\gamma+1}}{-\gamma+1}\right|_{V_3}^{V_4}$$

$$P.V^\gamma = cte = A$$

$$A =$$

$$W_{3\to4} = +\frac{1}{\gamma-1} A V^{-\gamma+1} \Big|_{V_3}^{V_4} = -\frac{PV}{\gamma-1}\Big|_3^4$$

$$= \frac{P_4V_4 - P_3V_3}{\gamma-1} = -4,48\times10^5 \text{ J}$$

$$W_{4\to1} = \int_4^1 \delta Q = -\int_4^1 P\,dV = -P_4(V_1-V_4) = 1,21\times10^5 \text{ J}$$

$$W_{cycle} = -4,19\times10^5 \text{ J} = -419 \text{ kJ}$$

Au cours du cycle l'air ne reçoit de la chaleur que pendant l'échauffement isobare

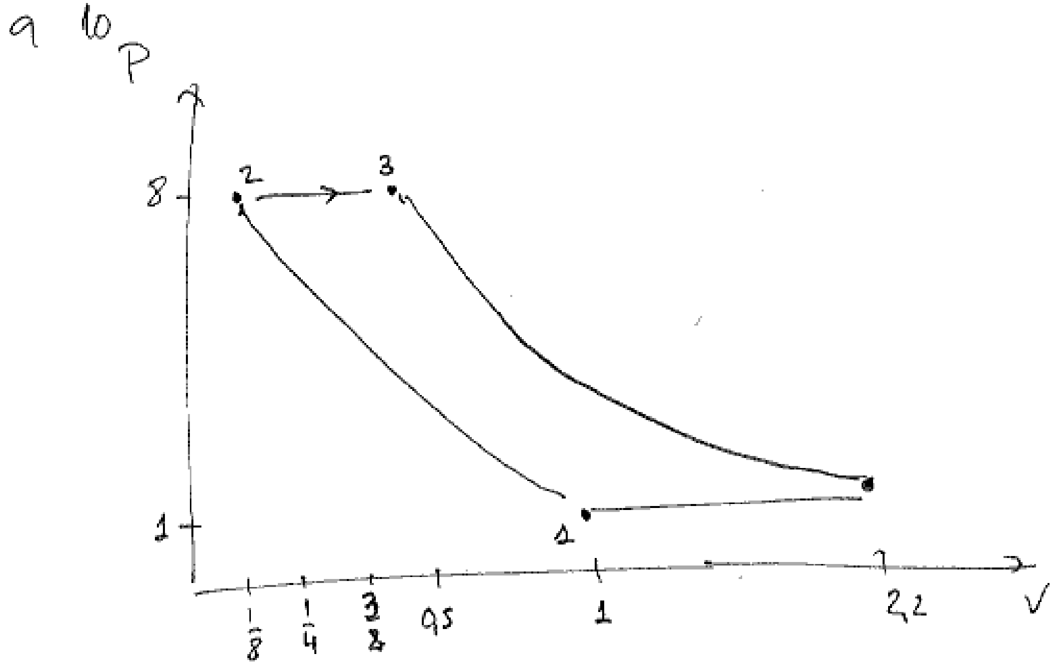
$$Q_{2\to3} = \int_2^3 \delta Q = \int_2^3 dH = n c_p (T_3 - T_2)$$

$$= 1,050\times10^6 \text{ J} \text{ soit } 1050 \text{ kJ}$$

$$\eta = \frac{|W|}{Q_{2\to3}} = 0,40$$

Le rendement de Carnot

$$\eta_c = 1 - \frac{T_1}{T_2} = 1 - \frac{350}{1400} = \underline{0,75}$$



Exercice 4

État A
$P_A = P_B$
$V_A = V_C$

$\rightleftarrows$

État B
$P_B = 1\times10^5$ Pa
$V_B = 2,00\times10^{-3}$ m^3
$T_B = 350$ K

compression adiabatique $\rightarrow$

État C
$P_C = 1,97\times10^6$ Pa
$V_C = V_B/8,4$
$T_C = 820$ K

isochore $\downarrow$

État D
$P_D = 6,77\times10^6$ Pa
$V_D = V_C$
$T_D = T_C + 2000$

$\leftarrow$ détente adiabatique

État E
$P_E = 3,44\times10^5$ Pa
$V_E = V_B$
$T_E = 1204$ K

isochore $\uparrow$

① $n = \dfrac{P_B \cdot V_B}{RT_B} \Rightarrow m = n\times M = \dfrac{P_B V_B}{RT_B}\times M = 2,00$ g

Transformation B → C adiabatique

$$P_B \times V_B^{\gamma} = P_C V_C^{\gamma}$$

$$P_C = P_B \left(\frac{V_B}{V_C}\right)^{\gamma}$$

$$P_C = 1{,}968 \times 10^{6}\ Pa$$

$$V_C = \frac{V_B}{8{,}4} = 0{,}24 \times 10^{-3}\ m^3$$

$$P_B^{\frac{1-\gamma}{\gamma}} T_B = P_C^{\frac{1-\gamma}{\gamma}} T_C$$

$$T_C = T_B \left(\frac{P_B}{P_C}\right)^{\frac{1-\gamma}{\gamma}}$$

$$T_C = 820\ K$$

Transformation isochore C → D

$$\frac{P_C}{T_C} = \frac{P_D}{T_D} \qquad P_D = P_C\ \frac{T_D}{T_C}$$

$$P_D = 6{,}768 \times 10^{6}\ Pa$$

Transformation adiabatique D → E

$$P_D V_D^{\gamma} = P_E V_E^{\gamma}$$

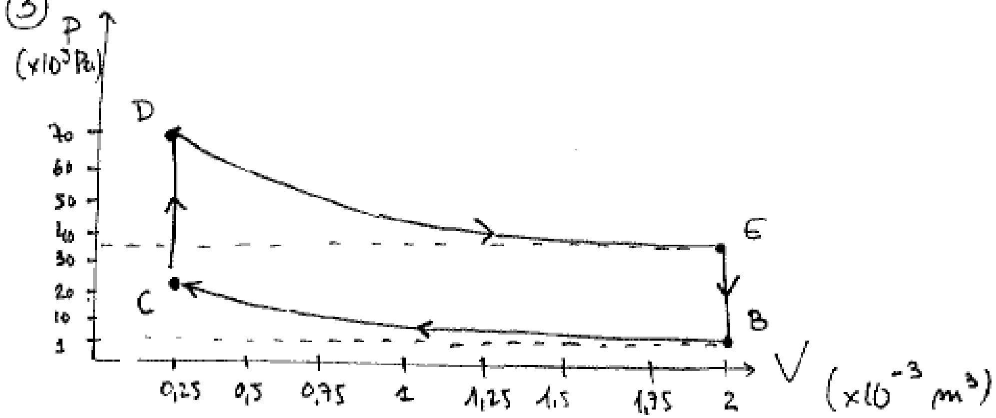
$$P_E = P_D \left(\frac{V_D}{V_E}\right)^{\gamma} \qquad P_E = 3{,}44 \times 10^{5}\ Pa$$

$$P_E^{\frac{1-\gamma}{\gamma}} \cdot T_E = P_D^{\frac{1-\gamma}{\gamma}} T_D$$

$$T_E = \left(\frac{P_D}{P_E}\right)^{\frac{1-\gamma}{\gamma}} T_D$$

$$T_E = 1204\ K$$

3



4 (a) Chaleurs échangées.

Transformations adiabatiques $\begin{cases} B \to C & Q_{B\to C} = 0 \\ D \to E & Q_{D\to E} = 0 \end{cases}$

Transformations isochores $\delta W = 0$

$$Q_{C\to D} = \int_C^D \delta Q = \int_C^D m c_v' \, dT = m c_v' (T_D - T_C)$$

$$= m \frac{c_p'}{\gamma} (T_D - T_C)$$

$$= 2857\ J$$

13 14

$$Q_{E\to B} = m\frac{c_p'}{\gamma}(T_B - T_E) = -1220\ J$$

$$\eta = \frac{-W}{Q_{C\to D}} = \underline{0,57}$$

<u>Travaux</u>

Transformations isochores $W_{E\to B} = W_{C\to D} = 0\ J$

Transformations adiabatiques.

$$W_{B\to C} = \int_B^C \delta W = \int_B^C dU = \int_B^C m c_v' dT =$$

$$= m\frac{c_p'}{\gamma}(T_C - T_B) = 671,3\ J$$

$$W_{D\to E} = m\frac{c_p'}{\gamma}(T_E - T_D) = -2308\ J$$

(b) $W_{cycle} = W_{B\to C} + W_{C\to D} + W_{D\to E} + W_{E\to B}$

$$= \underline{-1,637\times10^3\ J}$$

(c) Un cycle représente 2 tours

3000 tours représentent 1500 cycles

$$P = \frac{E}{t} = \frac{1500\times1,637\times10^3}{60} = 4,093\times10^4\ W$$

(d) $Q_{cycle} = Q_{B\to C} + Q_{C\to D} + Q_{D\to E} + Q_{E\to B} = 1637\ J$

La chaleur reçue par le mélange est $Q_{C\to D}$